Name: Solutions & Marking Scheme 2009 TG: PF

Unit 1 Astronomy: Kepler's Laws Of Planetary Motion – Assessed Activity ***(66 marks total)***

Aim: To investigate Kepler's three laws planetary motion.

Apparatus: Graph paper, 360^{o} protractor and ruler.

Data: The heliocentric data supplied is for the planet Mercury from 24th April 2008 to 21st July 2008.

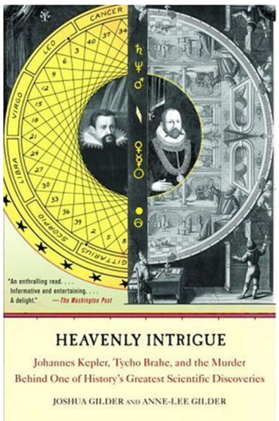
This activity has been divided into four parts:

Part A	Plotting and analysing planetary orbital data.	(10 marks)
Part B	Investigating Kepler's First Law.	(30 marks)
Part C	Investigating Kepler's Second Law.	(9 marks)
Part D	Investigating Kepler's Third Law.	(17 marks

Johannes Kepler (1571-1630)

Johannes Kepler was born on December 27th 1571. He was introduced to astronomy at an early age, and he developed a life long love for it. At the age of six Kepler observed the Great Comet of 1577 and wrote that he "was taken by his mother to a high place to look at it." At age nine, he observed another astronomical event, the Lunar eclipse of 1580, and recorded that he remembered being "called outdoors" to see it and that the moon "appeared quite red".

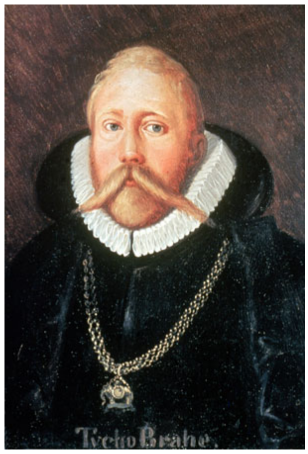
Unfortunately, childhood smallpox left Kepler with weak vision and crippled hands, severely limiting his ability in the observational aspects of astronomy during later life. In 1589 Kepler attended university and showed that he was a very talented mathematician. He adopted Copernicus' view of a heliocentric universe and defended his view theoretically and theologically. Kepler also maintained that the Sun was the source of the universes motive power.



In early 1600 Kepler established a working relationship with Tycho Brahe.

Tycho was an exceptional observational astronomer who spent many years, night after night, observing and carefully recording the motion of the planets and the positions of the stars. All without the aid of a telescope.

Although Tycho allowed Kepler to analyse the collected data he jealously guarded it and would not allow Kepler to copy it. This led to a great deal of friction and arguing between the pair.



Tycho Brahe (1546-1601)

After Tycho's death in 1601 Kepler illegally obtained Tycho's data and used it investigate and analyse the motion of Mars. After a number of failed attempts trying to fit the data to a circle Kepler decided to test if the data would fit an ellipse. The rest, as they say, is history.

Recent investigations suggest that Tycho died from mercury poisoning and not from urinary problems. Joshua Gilder and Anne-Lee Gilder in their 2005 book 'Heavenly Intrigue' put forward that there is substantial circumstantial evidence to suggest that Kepler murdered Brahe; they argue that Kepler had the means, motive, and opportunity, and that he had stolen Tycho's data on his death.

Part A Plotting and analysing planetary orbital data.

The supplied data is for the planet Mercury.

Date	RA (hrs)	Distance (AU)
24/4/08	4.933	0.308
28/4/08	6.817	0.312
2/5/08	8.583	0.325
6/5/08	10.067	0.345
10/5/08	11.267	0.367
14/5/08	12.267	0.391
18/5/08	13.133	0.412
22/5/08	13.933	0.431
26/5/08	14.700	0.446
30/5/08	15.467	0.457
3/6/08	16.217	0.464
7/6/08	17.017	0.467
11/6/08	17.833	0.465
15/6/08	18.683	0.458
19/6/08	19.567	0.448
23/6/08	20.467	0.433
27/6/08	21.417	0.415
1/7/08	22.400	0.394
5/7/08	23.417	0.371
9/7/08	0.517	0.348
13/7/08	1.767	0.328
17/7/08	3.233	0.313
21/7/08	4.950	0.308

Date	Angle (^{o})	Distance (cm)
24/4/08	90	6.9
28/4/08	118	7.0
2/5/08	145	7.3
6/5/08	167	7.7
10/5/08	185	8.2
14/5/08	200	8.8
18/5/08	213	9.2
22/5/08	225	9.7
26/5/08	237	10.0
30/5/08	248	10.2
3/6/08	259	10.4
7/6/08	271	10.5
11/6/08	283	10.4
15/6/08	296	10.3
19/6/08	310	10.0
23/6/08	323	9.7
27/6/08	337	9.3
1/7/08	352	8.8
5/7/08	7	8.3
9/7/08	24	7.8
13/7/08	43	7.3
17/7/08	64	7.0
21/7/08	90	6.9

In the above table the angles of Right Ascension (hrs) and the Orbital Distance (AU) have been converted into degrees and centimetres respectively so that they can be plotted onto graph paper.

Question 1

On the graph paper supplied, plot and label each position of Mercury's orbit using the values for angle and distance provided in the table on the previous page.

- heading (1 mark)
- labelling of plotted points (1 mark)
- accuracy of plotted points (2 marks)

(4 marks)

Question 2

Use a pencil to connect each of the plotted points in Question 1 with a smooth curve. Also indicate on your plot the direction of Mercury's orbit around the Sun.

- accuracy and neatness of curve (1 mark)
- direction of orbital path clearly shown (1 mark)

(2 marks)

Question 3

From the supplied data calculate the length of Mercury's year.

- each interval is 4 days in length. ∴ Mercury's year = 4 x 22 days (1 mark)
- there are 22 complete intervals in one orbit = 88 days (1 mark)

Also accept a calculation of 88 days based on the calendar dates provided.

(2 marks)

Question 4

What is an AU? How many kilometres are there in 1.00 AU?

- An astronomical unit, AU, is equal to the average distance between the Earth and the Sun. (1 mark)
- 1 AU = 149 598 000 km or 1.50 x 108 km (3sf) (1 mark)

(2 marks)

Part B Investigating Kepler's First Law.

Use your answers from Part A to answer Questions 5 to 11.

Question 5

What is the name of the shape that Mercury's orbit traces out as it travels around the Sun?

As Mercury orbits the Sun the path it traces out is an *ellipse*. (1 mark)

(1 mark)

Question 6

Using the symmetrical properties of the shape you have drawn, locate and mark on your graph paper, the second focal point of Mercury's orbit. Remember that the Sun is at the other focal point.

- 2nd focal point correctly located (1 mark)
- 2nd focal point labelled (1 mark)

(2 marks)

Question 7

Does Mercury travel at a constant speed as it orbits the Sun? Explain your answer.

- Mercury does not travel at a constant speed in its orbit around the Sun. (1 mark)
- If Mercury was travelling at a constant speed around the Sun then the distance between consecutive plotted points would be constant. From Question 1 it is clear that they are not. (1 mark)

(2 marks)

Question 8

If Mercury doesn't travel at a constant speed around its orbit, during which four day period is it travelling at its fastest? How would you describe Mercury's position with respect to the Sun during this four-day period?

- Mercury was travelling fastest between the 22nd and 26th April (ie on the 24th) (1 mark)
- During this interval (on this date) Mercury was closest to the Sun (1 mark)

Also accept 19th to 23rd July (ie on 21st).

(2 marks)

Question 9

If Mercury doesn't travel at a constant speed around its orbit, during which four day period is it travelling at its slowest? How would you describe Mercury's position with respect to the Sun during this four-day period?

- Mercury was travelling slowest between the 5th and 9th June (ie on the 7th) (1 mark)
- During this interval (on this date) Mercury was furthest from the Sun (1 mark)

(2 marks)

Question 10

With the aid of your textbook and/or the Internet, look up the words perihelion and aphelion and write down their meanings. Sketch a diagram of Mercury's orbit around the Sun and label the points of perihelion and aphelion. What is the special feature that connects these two points in Mercury's orbit and the Sun?

- Perihelion is the *distance* of closest approach an object has to the body it is orbiting. (1 mark)

- Aphelion is the furthest distance an object gets from the body it is orbiting. (1 mark)

Special Feature

- A straight line joining the points of perihelion and aphelion will pass through the Sun. (1 mark)
- This line is the major axis of Mercury's elliptical orbit and one of its axes of symmetry. (1 mark)

(4 marks)

Question 11

Using the supplied information and your answers to Part A calculate Mercury's average orbital distance from the Sun in AU's.

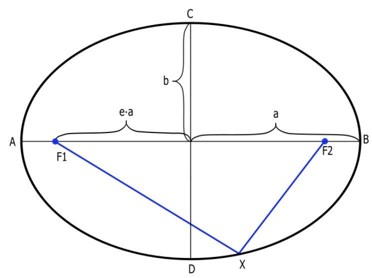
= (0.308 + 0.467)/2 (1 mark)

= 0.388 AU (1 mark) http://www.solarviews.com/eng/data.htm

(2 marks)

Question 12

What is meant by the term eccentricity with respect to a planetary orbit? How is an orbit's eccentricity calculated? What is the accepted value for the eccentricity of Mercury's orbit? Use measurements from your graph of Mercury's orbit to calculate its eccentricity and compare it to the accepted value. Comment on your findings.



- By definition a circle has an eccentricity, $\varepsilon = 0.000$. An ellipse has an eccentricity between 0.00 and 1.00. Eccentricity is a measure of 'how close' a closed curve is to a circle. (1 mark)

- The eccentricity of a planetary object's orbit can be calculated using the following rule: $\varepsilon = \frac{\sqrt{a^2 - b^2}}{a}$. (1 mark)

- From Question 1 $a = \frac{175}{2}$ mm and $b = \frac{171}{2}$ mm

  $a = 87.5$ mm and $b = 85.5$ mm

  (1 + 1 = 2 marks)

- So: $\varepsilon = \frac{\sqrt{87.5^2 - 85.5^2}}{87.5}$ (1 mark)

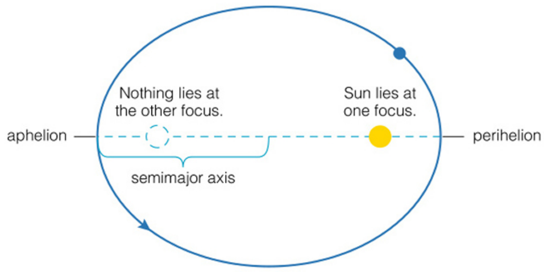
  $\varepsilon = 0.213$ (1 mark)

- This value compares very well, within 5%, of the documented value of $\varepsilon = 0.206$ (http://www.solarviews.com/eng/data.htm) (1 mark)

(7 marks)

Question 13

In point form write a paragraph that summarises the five key features of Mercury's orbit around the Sun. Include an appropriately labelled diagram as part of your answer.



http://www.bramboroson.com/astro/images/ellipse2.jpg

- Aphelion and perihelion (1 mark)
- Sun and second focal point (1 mark)
- Direction of orbit shown (1 mark)

- Mercury orbits the Sun in an anti-clockwise direction. (1 mark)
- Mercury orbits the Sun in an elliptical orbit with the Sun at one of its focal points. (1 mark)
- The length of Mercury's year is 88 days. (1 mark)
- At perihelion Mercury is at its closest to the Sun and travelling at its fastest. (1 mark)
- At aphelion Mercury is at its furthest from the Sun and travelling at its slowest. (1 mark)

(8 marks)

Part C Investigating Kepler's Second Law

To further describe planetary motion Kepler also stated that 'The line joining the planet to the Sun sweeps out equal areas in equal intervals of time'. In this part of the activity you will examine Kepler's second law.

Question 14

Work out the area that the line joining Mercury to the Sun sweeps out from April 28th to May 2nd.

There are two methods students can use to calculate an approximate value for the area swept out in a given time.

Counting Squares
(see graph)

A = 293 squares (1 mark)

$A_{\text{one square}} = 0.2 \times 0.2\ \text{cm}^2$

$A_{\text{one square}} = 0.04\ \text{cm}^2$ (1 mark)

A = 293 x 0.04 cm^2

A = 11.72 cm^2 (1 mark)

Allow $\pm 0.50\ \text{cm}^2$ for full marks.

Calculating Area
(see graph)

$A = \frac{1}{2} \times 6.9 \times 3.4 + \frac{1}{2} \times 3.4 \times 0.2\ \text{cm}^2$ (1 mark)

$A = 11.73 + 0.34\ \text{cm}^2$ (1 mark)

$A = 12.07\ \text{cm}^2$ (1 mark)

or

$A = \frac{\theta^o}{360^o} \pi r^2$

$A = \frac{26.5^o}{360^o} \pi \left(\frac{7.0+7.3}{2}\right)^2$ (2 marks)

$A = 11.82\ \text{cm}^2$ (1 mark)

(3 marks)

Question 15

Work out the area that the line joining Mercury to the Sun sweeps out from June 15th to June 19th.

Counting Squares
(see graph)

A = 306 squares (1 mark)

$A_{\text{one square}} = 0.2 \times 0.2\ \text{cm}^2$

$A_{\text{one square}} = 0.04\ \text{cm}^2$ (1 mark)

A = 206 x 0.04 cm^2

A = 12.00 cm^2 (1 mark)

Allow $\pm 0.50\ \text{cm}^2$ for full marks.

Calculating Area
(see graph)

$A = \frac{1}{2} \times 10 \times 2.4 + \frac{1}{2} \times 2.4 \times 0.1\ \text{cm}^2$ (1 mark)

$A = 12.00 + 0.12\ \text{cm}^2$ (1 mark)

$A = 12.12\ \text{cm}^2$ (1 mark)

or

$A = \frac{\theta^o}{360^o} \pi r^2$

$A = \frac{13.5^o}{360^o} \pi \left(\frac{10.3+10.0}{2}\right)^2$ (2 marks)

$A = 12.14\ \text{cm}^2$ (1 mark)

(3 marks)

Question 16

How do your answers to Questions 14 and 15 compare? Do they support Kepler's second law? Explain your answer.

- The answers obtained in Questions 14 and 15 are very close to each other. There is less than 3% difference between them. (1 + 1 = 2 marks)
- Based on this 'closeness' the results *do* support Kepler's 2nd Law that states: 'A line joining a planet to the Sun sweeps out equal areas in equal time.' (1 mark)

(3 marks)

Part D Kepler's Third Law

Kepler also found that for a planetary system the ratio $\frac{R^3}{T^2}$ was a constant value for each object orbiting the system's central body. For example the planets in our Solar System orbiting the Sun.

The following table provides data on the length of the year and average distance from the Sun for each of the planets in our Solar system.

Planet	Planetary Year Measured In Earth Days	Planet's Average Distance From Centre Of The Sun (km)	T^2	R^3	$\frac{R^3}{T^2}$
Mercury	88.00	5.80E+07	7.74E+03	1.95E+23	2.52E+19
Venus	224.70	1.08E+08	5.06E+04	1.27E+24	2.50E+19
Earth	365.25	1.50E+08	1.33E+05	3.35E+24	2.51E+19
Mars	686.90	2.28E+08	4.72E+05	1.18E+25	2.51E+19
Jupiter	4,331.87	7.78E+08	1.88E+07	4.71E+26	2.51E+19
Saturn	10,760.27	1.43E+09	1.16E+08	2.91E+27	2.51E+19
Uranus	30,681.00	2.87E+09	9.41E+08	2.36E+28	2.51E+19
Neptune	60,193.20	4.50E+09	3.62E+09	9.10E+28	2.51E+19

Question 17

By entering the above data into an excel spreadsheet, or by using your calculator, complete each of the three remaining columns in the above table. Give your answers correct to three significant figures (3sf).

- 1 mark for each column of correctly calculated data (3 x 1 = 3 marks)
- 1 mark for data correctly given to 3sf (1 mark)

(4 marks)

Question 18

Explain whether or not your answers to Question 17 support Kepler's 3rd Law.

- R^3/T^2 is approximately the same value for each our solar system's eight planets. (1 mark)
- Since R^3/T^2 is constant for each of the planets the supplied data does support Kepler's 3rd Law. For the planets orbiting the Sun in our solar system the value of $R^3/T^2 = 2.51 \times 10^{19}$. (1 + 1 = 2 marks)

(3 marks)

The data table on the next page shows data for the moons orbiting the planets Jupiter and Saturn.

Planet	Satellite (Moon)	Satellite Year Measured In Earth Days	Average Distance Of Moon From Its Planet (km)	$\frac{R^3}{T^2}$
Jupiter	J15 Adrastea	0.30	1.34E+05	2.67E+16
	V Amalthea	0.49	1.81E+05	2.47E+16
	XIV Thebe	0.67	2.22E+05	2.44E+16
	I Io	1.77	4.22E+05	2.40E+16
	II Europa	3.55	6.71E+05	2.40E+16
	III Ganymede	7.15	1.07E+06	2.40E+16
	IV Callisto	16.70	1.88E+06	2.38E+16
	XIII Leda	240.00	1.11E+07	2.38E+16
	VI Himalia	251.00	1.15E+07	2.40E+16
	X Lysithea	260.00	1.17E+07	2.38E+16
	VII Elara	260.00	1.17E+07	2.39E+16
	XII Anake	617.00	2.07E+07	2.33E+16
	XI Carme	692.00	2.24E+07	2.33E+16
	VIII Pasiphae	735.00	2.33E+07	2.34E+16
	IX Sinope	758.00	2.37E+07	2.32E+16
Saturn	Atlas	0.22	1.37E+05	5.31E+16
	1980 S27	0.61	2.79E+05	5.81E+16
	1980 S26	0.63	2.83E+05	5.73E+16
	Janus	0.69	3.03E+05	5.83E+16
	Epimetheus	0.69	3.03E+05	5.84E+16
	Mimas	0.94	3.71E+05	5.77E+16
	Enceladus	1.37	4.76E+05	5.76E+16
	Tethys	1.89	5.89E+05	5.73E+16
	Dione	2.74	7.55E+05	5.73E+16
	1980 S6	2.74	7.55E+05	5.73E+16
	Rhea	4.42	1.05E+06	5.99E+16
	Titan	15.95	2.44E+06	5.74E+16
	Hyperion	21.28	2.96E+06	5.75E+16
	Iapetus	79.33	7.12E+06	5.75E+16
	Phoebe	550.33	2.59E+07	5.75E+16

Question 19

Choose the moons from either Jupiter or Saturn. You do not have to do both. By entering the above data into an excel spreadsheet, or by using your calculator, complete the remaining column in the above table. Give your answers correct to three significant figures (3sf).

- 1 mark for a column of correctly calculated data (1 mark)
- 1 mark for data correctly given to 3sf (1 mark)

(2 marks)

Question 20

Explain whether or not your answer to Question 19 also supports Kepler's 3rd Law. What do you notice when you compare the average value for $\frac{R^3}{T^2}$ in Question 17 to that of Question 19? Try and explain your findings.

- For Jupiter's moons the average value of $R^3/T^2 = 2.50 \times 10^{16}$. (1 mark)
- For Saturn's moons the average value of $R^3/T^2 = 5.75 \times 10^{16}$. (1 mark)

- The value of R^3/T^2 for Jupiter's (Saturn's) moons is also constant and therefore supports Kepler's 3rd Law. (1 mark)
- However, its value is different compared to that obtained for the solar system. (1 mark)

This difference is due to the central body (Sun/Planet) of the system. (1 mark)

(4 marks)

Conclusion:

Write a summary of your findings from the four parts of this activity.

- A planetary object will orbit a larger central body in an elliptical path with the central body at one of the ellipse's focal points. (Kepler's 1st Law) (1 mark)

- A line joining an orbiting body with its central body will always sweep out equal area in equal time. (Kepler's 2nd Law) (1 mark)

- The value of R^3/T^2 is constant for all bodies orbiting the same central body. (Kepler's 3rd Law) (1 mark)

- The value of R^3/T^2 varies with a change in central body. (Kepler's 3rd Law) (1 mark)

(4 marks)

Kepler's Laws

http://hyperphysics.phy-astr.gsu.edu/hbase/kepler.html